Fitting of Curves

(1) Best fitted straight line

$$\boxed{y = a + bx}$$

(2) Best fitted Parabola

$$\boxed{y = a + bx + cx^2 \,;\ c \neq 0}$$

(3) Exponential curve

$$\boxed{y = ae^{bx}}$$

Fitting

Let us consider the exponential curve of the form

$$\boxed{y = ae^{bx}} \quad \text{——— (1)}$$

where a and b are arbitrary constants to be determined on the [illegible]

... the basis of paired values of x and y

Taking log on both sides of equation we get

$$\log y = \log (ae^{bx})$$

$$\log_e (mn) = \log_e m + \log_e n \quad \text{— (i)}$$
$$\log_e \left(\frac{m}{n}\right) = \log_e m - \log_e n \quad \text{— (ii)}$$
$$\log_e m^n = n \cdot \log m \quad \text{— (iii)}$$

$$\log y = \log_e a + \log e^{bx}$$
$$= \log_e a + bx \cdot \log_e e$$

$$\Rightarrow \log y = \log_e a + bx$$

Now let

$$\log_e y = Y, \quad \log_e a = A$$

$b = B$, $x = X$

$$\Rightarrow \quad \boxed{Y = A + BX}$$

By using least square principle we get the normal equations

$$\sum Y_i = nA + B\sum_i X_i \quad \text{—— (A)}$$

$$\sum_i X_i Y_i = A\sum_i X_i + B\sum_i X_i^2 \quad \text{—— (B)}$$

on obtaining the values of A and B, we obtain the values of a and b

$$a = e^A = \text{antilog}(A)$$
$$b = B$$

on inserting these values in equation (1), we get the best fitted exponential curve and having the following form

$$\boxed{\hat{y} = \hat{a} e^{\hat{b} x}}$$

which provides us required best fitted exponential curve on the paired observations x and y.

Problem

For the following data

x	1	2	3	4
y	2	3	4	5

Fit the best fitted exponential curve.

Soln We know that exponential curve having the following form

$$y = a e^{bx} \quad \text{--- (1)}$$

where a and b are arbitrary constants to be determined on the basis of paired observations.

$$\log_e y = \log_e a + bx \cdot \log_e e$$

$$\Rightarrow \log_e y = \log_e a + bx$$

$$\Rightarrow \boxed{Y = A + BX} \quad \text{—— (2)}$$

where $Y = \log_e y$, $A = \log_e a$

$b = B$, $x = X$

$$\sum Y_i = nA + B \sum X_i \quad \text{—— (i)}$$

$$\sum_i Y_i X_i = A \sum_i X_i + b \sum X_i^2 \quad \text{—— (ii)}$$

Now, we have to prepare the following observation table to get the required values as follows

Table

x	y	$X = x$	$Y = \log_e y$	XY	X^2
1	2	1	$\log_e 2 =$	→	1

2	3	2	$\log_e 3 =$	$\rightarrow$	4
3	4	3	$\log_e 4$	$\rightarrow$	9
4	5	4	$\log_e 5$	$\rightarrow$	16
Total		$\Sigma x_i = 10$	$\Sigma y_i = 4.70$	$\Sigma x_i y_i = 13.40$	$\Sigma x_i^2 = 30$

So, the normal equations will be

$$4.70 = 4A + 10B$$
$$13.40 = 10A + 30B$$

on solving these,

$$A = 0.43$$
$$B = 0.306$$

So, $a = \text{antilog } A = e^{A} = e^{0.43}$

$$\Rightarrow a = 1.5372$$
$$b = B = 0.306$$

on putting these values of a and b in equation (1), we get the best fitted exponential curve is

$$y = 1.5372 \cdot e^{0.306x}$$