

Udai Pratap (Autonomous) College, Varanasi**E-learning Material**

Module/ Lecture	02
Topic	Transport Phenomena
Developed by	<i>Dr. Devendra Kumar Singh</i> <i>Assistant Professor,</i> <i>Physics Department,</i> <i>Udai Pratap (Autonomous) College, Varanasi</i>

Transport Phenomena

The equilibrium state of a system is most probable state of the system. In this state the average values of the macroscopic quantities such as density, pressure, temperature, momentum of molecules, their energies etc remains same everywhere. But if the system is not in equilibrium state, any of the following three cases singly or jointly may occur –

1. The different parts of the gas may have different molecular velocities; this will result in the relative motion of the different layers of the gas with respect to one another. In this case layer moving faster will impart momentum to the layers moving slower to bring the gas in equilibrium state. This gives rise to the phenomenon of viscosity.
2. The different parts of the gas may have different temperature. In such a case, the molecule of the gas will carry kinetic energy from regions of higher temperature to the region of lower temperature to attain the equilibrium, resulting in transport of energy. This give rise to the phenomenon of thermal conductivity.
3. The different parts of the gas may have different molecular concentration (i.e. the number of molecules per unit volume). In such a case molecule of the gas will move from region of higher concentration to region of lower concentration resulting in the transport of mass. This gives rise to the phenomenon of diffusion.

We thus find that viscosity, conduction and diffusion respectively represent the transport of momentum, energy and mass. Hence these phenomena are called transport phenomenon.

1. Viscosity

Consider a gas at constant temperature T moving in x -direction with a velocity which is increasing with z -direction. Since

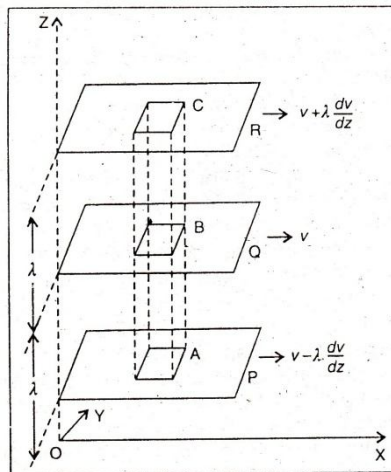
- i. a molecule changes its direction only on collision and
- ii. it will move an average distance λ (mean free path) before it undergoes collision.

Therefore the motion of the gas can be assumed to occur in parallel layers such that adjacent layers with different velocities are separated by distance λ .

Let **Q** be an arbitrary layer with uniform velocity v in x -direction and a velocity gradient dv/dz . The velocity of layer **R** above it will be $v + \lambda(dv/dz)$ and that of layer **P** below it will be $v - \lambda(dv/dz)$.

A molecule of mass μ reaching layer **Q** from layer **R** bring horizontal momentum $p^R = \mu [v + \lambda(dv/dz)]$.

Similarly, a molecules reaching layer **Q** from layer **P** bring horizontal momentum $p^P = \mu [v - \lambda(dv/dz)]$.



Draw a cylinder **BC** of height λ and until area of cross section between layer **Q** and **R**. It's volume will be λ and will contain $n_v \lambda$ molecules, where n_v is the number of molecules per unit volume.

Since the molecules of the gas execute random motion due to its thermal state. All the directions are equally possible for them.

Thus $(1/6)n_v\lambda$ molecules are moving downward direction with a average speed \bar{c} . Also the molecules will take time λ/\bar{c} travel from R to P; i.e. in λ/\bar{c} time $(1/6)n_v\lambda$ are crossing downward direction.

Thus the number of molecules crossing B in downward direction per second is

$$\frac{(1/6)n_v\lambda}{\lambda/\bar{c}} = \frac{1}{6}n_v\bar{c}$$

The same number will cross Q in upward direction which can be evaluated by drawing the cylinder AB between layers P and Q.

Since there is no accumulation of molecules in the layer B, therefore, the **net momentum transformed** to this layer **per unit area per second** in downward direction,

$$\begin{aligned} &= [\mu\{v+\lambda(dv/dz)\}(1/6)n_v\bar{c} - \mu v(1/6)n_v\bar{c}] - [\mu\{v-\lambda(dv/dz)\}(1/6)n_v\bar{c} - \mu v(1/6)n_v\bar{c}] \\ &= \frac{1}{3}n_v\bar{c}\mu\lambda(dv/dz) \end{aligned}$$

But according to Newton's law of motion, the rate of change of momentum is equal to force, therefore the force acting on the unit are of layer Q is

$$F = \frac{1}{3}n_v\bar{c}\mu\lambda(dv/dz) \quad (1)$$

The force will act tangentially on the layer. According to definition of coefficient of viscosity,

$$F = \eta A(dv/dz) \quad (2)$$

comparing Eqs. (1) and (2) we get,

$$\eta = \frac{1}{3}\mu n_v\bar{c}\lambda$$

Since mean free path $\lambda = \frac{1}{\sqrt{2}n_v\sigma}$

Thus

$$\eta = \frac{\mu\bar{c}}{3\sqrt{2}\pi\sigma^2} \quad (3)$$

Where σ is the diameter of a molecule.

It's clear from above that

$$\eta \propto \bar{c} \propto \sqrt{T}$$

Also viscosity is independent of the pressure.

2. Thermal Conductivity

Let the temperature of the gas increases in the z-direction. Let us consider three layers P, Q and R of the gas separated by distance λ (=mean free path). Let the temperature of layer Q be T and temperature gradient be $\frac{dT}{dz}$. Then the temperature

of layers P and R will be $T - \lambda \frac{dT}{dz}$ and $T + \lambda \frac{dT}{dz}$ respectively.

Since the molecules in a gas execute random motion, they move equally in all directions.

Thus if there are n_v molecules per unit volume of the gas and \bar{c} be the average speed, then the average number of molecules crossing per unit area in per unit time in any direction will be $\frac{1}{6}n_v\bar{c}$.

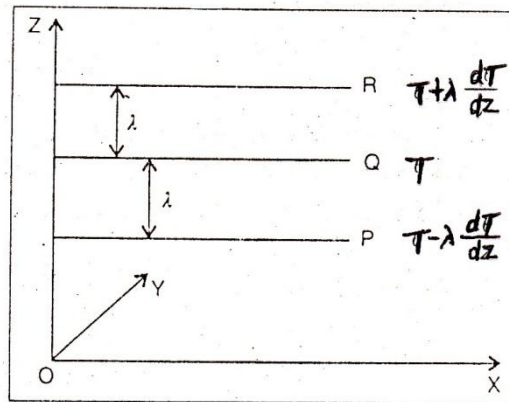
The mean energy transported across layer Q by molecules coming from layer R per unit area per unit time

$$Q^+ = \mu C_v \left(T + \lambda \frac{dT}{dz} \right) \frac{1}{6} n_v \bar{c} \quad (1)$$

Where C_v is molar specific heat.

Similarly, the mean energy transported across layer Q by molecules coming from layer P per unit area per unit time

$$Q^- = \mu C_v \left(T - \lambda \frac{dT}{dz} \right) \frac{1}{6} n_v \bar{c} \quad (2)$$



Therefore the net amount of heat crossing layer Q in downward direction per unit area per second is –

$$Q = Q^+ - Q^- = \frac{1}{3} \mu n_v \bar{c} C_v \lambda \frac{dT}{dz} \quad (3)$$

Now according to definition, the heat flow per second per unit area across the layer B in downward direction

$$Q = K \frac{dT}{dz} \quad (4)$$

Where K is known as coefficient of thermal conductivity.

Comparing Eqs. (1) and (2) we get

$$K = \frac{1}{3} \mu n_v \bar{c} C_v \lambda$$

Since mean free path $\lambda = \frac{1}{\sqrt{2} \pi n_v \sigma^2}$

Thus

$$K = \frac{1}{3} \mu n_v \bar{c} C_v \frac{1}{\sqrt{2} \pi n_v \sigma^2}$$

$$K = \frac{\mu \bar{c} C_v}{3\sqrt{2} \pi n_v \sigma^2} \quad (5)$$

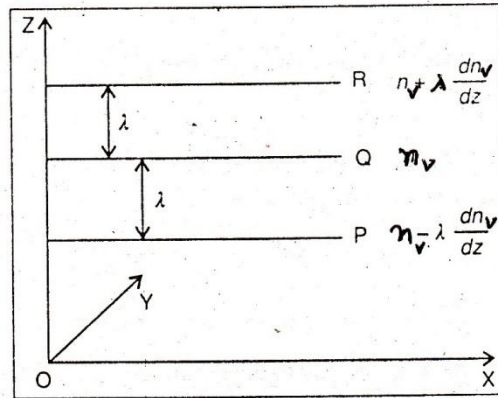
It's clear from above that

$$K \propto \bar{c} \propto \sqrt{T} \quad (6)$$

Also conductivity is independent of the pressure.

3. Gas self Diffusion

Suppose we have a concentration gradient in a gas increasing along z axis. Let n_v be the number of molecules per unit volume in layer Q and dn_v/dz be the concentration gradient the +ve z direction.



The number of molecules per unit volume in layers P and R will be $n_v - \frac{dn_v}{dz}$ and $(n_v + \frac{dn_v}{dz})$ respectively.

Due to the random motion of the molecules on an average one-sixth of the molecules will travel in any direction. If \bar{c} be the average speed of the molecules; the number of molecules reaching per unit area per unit time on layer Q from layer R (in downward direction) will be

$$n_v^+ = \frac{1}{6} \left(n_v + \lambda \frac{dn_v}{dz} \right) \bar{c} \quad (1)$$

Similarly the number of molecules reaching per unit area per unit time on layer Q from layer P (in upward direction) will be

$$n_v^- = \frac{1}{6} \left(n_v - \lambda \frac{dn_v}{dz} \right) \bar{c} \quad (2)$$

Thus the net number of molecules transported across layer Q in downward direction per unit area per second

$$\Gamma = n_v^+ - n_v^- = \frac{1}{3} \lambda \bar{c} \frac{dn_v}{dz} \quad (3)$$

Also the number of molecules crossing a given surface per unit area per unit time are given by Fick's law

$$\Gamma = D \frac{dn_v}{dz} \quad (4)$$

Where D is the coefficient of self diffusion.

Comparing Eqs. (3) and (4) we get

$$D = \frac{1}{3} \lambda \bar{c} \quad (5)$$

Since mean free path $\lambda = \frac{1}{\sqrt{2} \pi n_v \sigma^2}$

Thus

$$D = \frac{\bar{c}}{3\sqrt{2} \pi n_v \sigma^2} \quad (6)$$

Since $\lambda \propto \frac{1}{n_v} \propto \frac{T}{p}$ and $\bar{c} \propto \sqrt{T}$

Thus

$$D \propto T^{3/2} p^{-1}$$

i.e. coefficient of self diffusion depends on temperature and pressure both.