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Udai Pratap (Autonomous) College, Varanasi

E-learning Material

Module/ Lecture	**12**
Topic	**Planck's Law**
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Planck's Law

The earlier attempts to explain the energy distribution in the spectrum of black body are due to Wein's and Rayleigh Jeans. Wein's in 1893 explained it on the basis of classical thermodynamics whereas Rayleigh Jeans in 1900 explained it on the basis of Classical Electrodynamics theory with the application of statistical mechanics and equipartition of energy. These theories failed to give results compatible with experimental observations.

In order to explain the energy distribution in the spectrum of black body Max Planck in 1905 proposed a new revolutionary hypothesis known as quantum theory by means of which he was able to derive the correct law of energy distribution in the spectrum of black body.

Planck made the following two assumptions regarding the emission and absorption of radiations –

1. A black body radiation chamber is filled up not only with radiation but also with simple harmonic oscillator or rasonators (energy emitters) [known as planck's oscillators] of molecular dimensions. An oscillator cannot radiate or absorb energy continuously, but energy is emitted or absorbed descretely in the form of quanta called photons.

   An oscillator absorbs energy from the radiation field and deliver it back to the field in quanta of energy 0, ε, 2ε etc , where ε is the quantum of energy which is proportional to frequency υ of the oscillator i.e

   $$\varepsilon = h\upsilon \qquad (1)$$

   where h = planck's const = $6.61*10^{-34}$ j.s

2. The number of Planck's oscillators (resonators) emitting particular energy is given by statistical distribution law of Boltzmann.

According to this law, the number of oscillators with energy ε is proportional to $e^{-\varepsilon/kT}$.

If N be the total number of Planck's oscillators and E is their total energy, then the average energy per oscillator is given by

$$\bar{\varepsilon} = \frac{E}{N} \quad (2)$$

If $N_0, N_1, N_2, \ldots N_r \ldots$ be the number of oscillators having energies 0, ε, 2ε $r\in \ldots$ respectively we have

$$N = N_0 + N_1 + \ldots + N_r + \ldots \quad (3)$$

and

$$E = 0.N_0 + \varepsilon N_1 + 2\varepsilon N_{2} + \ldots r\varepsilon N_r + \quad (4)$$

According to Boltzmann distribution formula, the number of oscillators having $r\varepsilon$ will be

$$N_r = N_0 e^{-r\varepsilon/kT} \quad (5)$$

i.e.

$$N_1 = N_0 e^{-\varepsilon/kT},\ N_1 = N_0 e^{-\varepsilon/kT} \ldots\text{etc}$$

Thus from Eqs. (3 & 5), we get

$$N = N_0 + N_0 e^{-\varepsilon/kT} + N_0 e^{-2\varepsilon/kT} \ldots + N_0 e^{-r\varepsilon/kT} + \ldots$$

$$= N_0 (1 + e^{-\varepsilon/kT} + e^{-2\varepsilon/kT} \ldots + e^{-r\varepsilon/kT} + \ldots)$$

$$= N_0 \left(\frac{1}{1 - e^{-\varepsilon/kT}} \right) \quad (6)$$

from Eqs. (4 & 5), we get

$$E = 0.N_0 + \varepsilon N_0 e^{-\varepsilon/kT} + 2\varepsilon N_0 e^{-2\varepsilon/kT} + \ldots r\varepsilon N_0 e^{-r\varepsilon/kT} + \ldots$$

$$= N_0 \varepsilon (e^{-\varepsilon/kT} + 2 e^{-2\varepsilon/kT} + \ldots + r e^{-r\varepsilon/kT} + \ldots)$$

$$= N_0\varepsilon \frac{e^{-\varepsilon/kT}}{\left(1-e^{-\varepsilon/kT}\right)^2} \tag{7}$$

From Eqs. (2, 5, & 6), average energy of oscillator is

$$\bar{\varepsilon} = \frac{E}{N} = \frac{\left(N_0 \varepsilon e^{-\varepsilon/kT}\right) \Big/ \left(1-e^{-\varepsilon/kT}\right)^2}{N_0 \Big/ \left(1-e^{-\varepsilon/kT}\right)}$$

$$= \frac{\varepsilon}{\left(e^{\varepsilon/kT} - 1\right)}$$

i.e.

$$\bar{\varepsilon} = \frac{h\upsilon}{\left(e^{h\upsilon/kT} - 1\right)} \tag{8}$$

Thus according to Planck the average energy of an oscillator, depends not only upon the temperature (according to kinetic theory E = kT) but also upon the frequency υ .

Since the number of Plank's oscillators per unit volume in the frequency range υ and $\upsilon + d\upsilon$ (i. e. total no. of modes of vibrations between frequency range υ and $\upsilon + d\upsilon$ per unit volume) is $\frac{8\pi\upsilon^2}{c^3} d\upsilon$.

Thus the energy density between the frequency range υ and $\upsilon + d\upsilon$ is :

$$u_\upsilon d\upsilon = \frac{8\pi\upsilon^2}{c^3} d\upsilon \frac{h\upsilon}{\left(e^{h\upsilon/kT} - 1\right)}$$

$$u_\upsilon d\upsilon = \frac{8\pi h\upsilon^3}{c^3}\frac{1}{\left(e^{h\upsilon/kT} - 1\right)}d\upsilon \tag{9}$$

This is known as Planck's Radiation Law in terms of frequency. This law can also be represented in terms of wavelength as given below:

The energy density between the range of wavelength λ and $\lambda + d\lambda$ can be obtained by using ,

$$\upsilon = \frac{c}{\lambda}, \qquad d\upsilon = -\frac{c}{\lambda^2}d\lambda. \qquad \text{and} \qquad u_\lambda d\lambda = -u_\upsilon d\upsilon$$

$$-u_\lambda d\lambda = \frac{8\pi hc^3}{c^3\lambda^3}\frac{1}{\left(e^{hc/\lambda kT} - 1\right)}\left(-\frac{c}{\lambda^2}d\lambda\right)$$

Or
$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{\left(e^{hc/\lambda kT} - 1\right)}d\lambda \tag{10}$$

This is Planck's Radiation Law in terms of wavelength.

The Planck's law agrees well with the experimental results for all wavelengths. Furthermore the classical formulae can be deduced from it as special cases holding good under certain assumptions .Wein's and Rayleigh jean's law can be derived with the help of Planck's law as follows :-

Case-1: For Short Wavelengths

For short wavelengths $e^{hc/\lambda kT} \rangle\rangle 1$ and 1 can be neglected in the denominator of equation (10) and Planck's formula reduces to

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda kT} d\lambda \qquad (11)$$

This is Wein's law.

Case-2: For Long Wavelengths

For long wavelengths, $e^{hc/\lambda kT} \cong 1 + \frac{hc}{\lambda kT}$, so that Planck's formula (10) reduces to

$$u_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda \qquad (12)$$

This is Rayleigh jeans law.