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Udai Pratap (Autonomous) College, Varanasi

E-learning Material

Module/ Lecture	**04**
Topic	**Carnot's Heat Engine**
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Carnot's Heat Engine

The ideal heat free from all the imperfections of actual engine was supposed by Carnot for theoretical investigations. It consist of

1. A cylinder with perfectly non-conducting walls having a perfectly conducting bottom. The cylinder is closed with a perfectly non-conducting frictionless piston. The working substance used is a perfect gas to which cycle is applied.
2. **Source :** It is a hot reservoir maintained at constant high temperature T_1 from which working substances can draw heat . The top of it is perfect conductor.
3. **Sink :** It is a cold reservoir maintained at constant low temperature T_2 to which any amount of heat can be rejected . The top of this is also perfect conductor.
4. **Heat Insulating Stand :** It is a perfectly non conducting platform to serve as a stand for the cylinder.

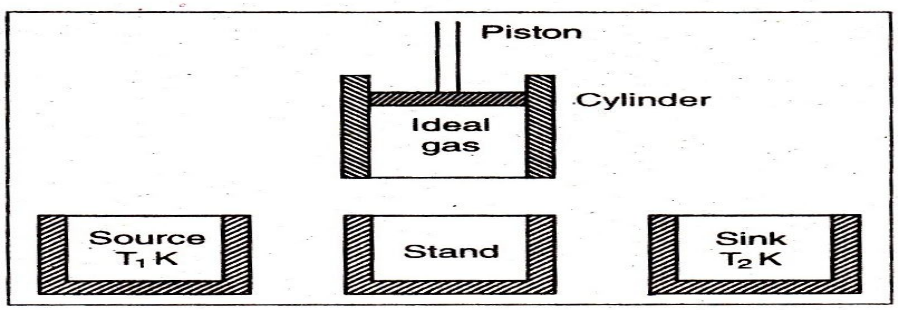

Fig:1

The Carnot Cycle: The Gas in the cylinder is subjected to a series of operations constituting a complete cycle as follows:

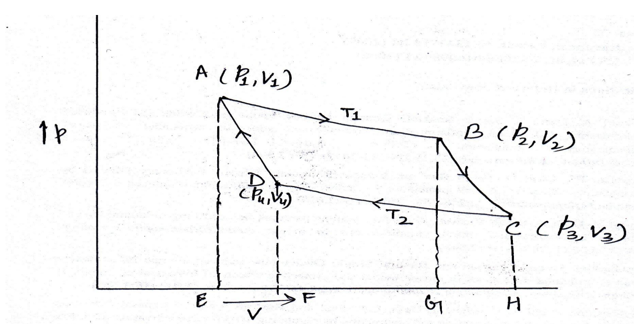

Fig : 2 Indicator Diagram

1. **Isothermal Expansion :** Let the cylinder containing the working substance be placed on the source at temp T_1. Let p_1 and V_1 be the pressure and volume of the gas represented by point A in the indicator diagram 1. Suppose the piston moves slowly outward so that gas expands, the temperature tends to fall , but as the cylinder is in contact with source , it will take necessary heat through the base by conduction and thus the gas expand isothermally at temp T_1 of the source until the pressure and volume become p_2 and V_2 represented by point B .

   Let Q_1 be the heat absorbed in the process $(A \rightarrow B)$, According to first law of thermodynamics,

   Q_1=work done by the gas in expanding isothermally from $A \rightarrow B$

   $$=W_1=\int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{nRT_1}{V} dV \qquad \text{(as } pV=nRT_1\text{)}$$

   $$= nRT_1 \log_e\left(\frac{V_2}{V_1}\right) \qquad (1)$$

2. **Adiabatic Expansion :** The cylinder is removed from the source and placed on the perfectly insulating stand and the piston is allowed to move further so that we get adiabatic expansion hence the temperature falls (working substance perform external work in raising the piston as the expense of the internal energy and hence temperature falls), the gas is now allowed to expand adiabatically till it temperature falls to T_2 , the temperature of sink.

   If p_3 and V_3 be the pressure and volume at C, then the work done by the gas from B to C is given by

   $$W_2=\int_{V_2}^{V_3} p\,dV$$

   For adiabatic process $pV^{\gamma} = K$ (Constant)

   $$W_2=K\int_{V_2}^{V_3} \frac{dV}{V^{\gamma}}$$

$$=\frac{K}{1-\gamma}\left(\frac{1}{V_3^{\gamma-1}}-\frac{1}{V_2^{\gamma-1}}\right)$$

$$=\frac{1}{1-\gamma}\left(\frac{KV_3}{V_3^{\gamma}}-\frac{KV_2}{V_2^{\gamma}}\right)$$

$$=\frac{1}{1-\gamma}\left(p_3V_3-p_2V_2\right)$$

Since pV=nRT

$$W_2=\frac{nR}{\gamma-1}\left(T_1-T_2\right) \tag{2}$$

The gas is now at relatively low pressure. To be used in a cyclic order, it has to be restored to its initial state. To get this effect, gas is compressed in two stages , first isothermally and then adiabatically as follows –

3. **Isothermal compression** – Place the cylinder on the sink at temperature T_2 and the gas is isothermally compressed until its pressure and volume becomes p_4 and V_4 represented by the points D.

The amount of heat rejected to the sink during process is equal to the work done W_3 on the gas

Q_2 =work done on the gas in compressing isothermally from $C \rightarrow D$

$$=W_3=\int_{V_3}^{V_4} p\,dV = \int_{V_1}^{V_2} \frac{nRT_2}{V}dV \qquad \text{(as } pV=nRT_2\text{)}$$

$$=nRT_2 \log_e\left(\frac{V_4}{V_3}\right)$$

$$=-nRT_2 \log_e\left(\frac{V_3}{V_4}\right) \tag{3}$$

4. **Adiabatic compression**: The cylinder is removed from the sink and placed again on the insulating stand . The gas is compressed adiabatically until the temperature rises to T_1 and gas attain its initial pressure and volume p_1, V_1 The work done on the gas during the process is -

$$W_4 = \int_{V_4}^{V_1} p\,dV$$

$$W_4 = -\frac{nR}{\gamma - 1}(T_1 - T_2) \qquad (4)$$

Net work done in one complete cycle

$$W = W_1 + W_2 + W_3 + W_4 = \text{Area ABCD}$$

Using Eqs. (1-4), we get

$$W = nRT_1 \log_e\left(\frac{V_2}{V_1}\right) - nRT_2 \log_e\left(\frac{V_3}{V_4}\right) \qquad (5)$$

Since points A and D lie on the same adiabatic

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \qquad (6)$$

Similarly points B and C lie on the same adiabatic, thus

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \qquad (7)$$

From Eqs. (6 & 7)

$$\frac{V_1}{V_4} = \frac{V_2}{V_3}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4} \qquad (8)$$

From Eqs. (5 & 8)

$$W = nR\,(T_1 - T_2)\log_e\left(\frac{V_2}{V_1}\right) \qquad (9)$$

Thus heat has been converted into work by the system and any amount of work can be obtained by simply repeating the cycle.

Thus efficiency of Carnot's heat engine

$$\eta = \frac{\text{Work Output}}{\text{Heat absorbed from source}} = \frac{W}{Q_1}$$

$$\eta = \frac{nR\,(T_1 - T_2)\log_e\left(\frac{V_2}{V_1}\right)}{nR\,T_1 \log_e\left(\frac{V_2}{V_1}\right)}$$

$$\eta = 1 - \frac{T_2}{T_1} \qquad (10)$$

Thus efficiency of carrot engine is independent of working substance and depend only on the temperature of the source and sink and is always less than unity since $T_1 > T_2$. The greater due difference between T_1 & T_2 the greater will be the efficiency.

Reversiblity of carnot's cycle

Carnot's cycle is perfectly reversible on account of –

1) Total absence of friction between the piston and the cylinder.
2) The extreme slowness of the operation performed on the working substance.
3) Prevention of loss of heat by conduction from the gas to the piston and cylinder by employing perfectly heat insulating Smaterial for the piston and walls of the cylinder.
4) Strictly constant temperature under which isothermal changes are made.